Bull. Korean Math. Soc. **0** (0), No. 0, pp. 1–0
https://doi.org/10.4134/BKMS.b190316
pISSN: 1015-8634 / eISSN: 2234-3016

ON THE ACTIONS OF HIGMAN-THOMPSON GROUPS BY HOMEOMORPHISMS

Jin Hong Kim

Abstract. The aim of this short paper is to show some rigidity results for the actions of certain finitely presented groups by homeomorphisms. As an interesting and special case, we show that the actions of Higman-Thompson groups by homeomorphisms on a cohomology manifold with a non-zero Euler characteristic should be trivial. This is related to the well-known Zimmer program and shows that the actions by homeomorphism could be very much different from those by diffeomorphisms.

1. Introduction

It is well-known that the special linear group $SL(n,\mathbb{Z})$ over integers acts effectively on the unit sphere S^{n-1} by using the linear action on the Euclidean space $\mathbb{R}^n$, while if $n \geq 3$, then it cannot act effectively on S^r for any $r < n-1$. More generally, it is believed that any action of $SL(n,\mathbb{Z})$, $n \geq 3$, on a compact connected manifold of dimension r less than $n-1$ by homeomorphisms factors through a finite group action. This is related to the well-known Zimmer program about group actions of lattices in Lie groups on manifolds that is one of the motivations of this paper (see [5] for more details).

When $r = 1$, this problem has been resolved in [10] by Witte. As another remarkable and positive result, Bridson and Vogtmann proved in [4] that if $n \geq 3$ and $r < n$, then $SL(n,\mathbb{Z})$ cannot act effectively by homeomorphisms on any contractible manifold of dimension r and on any homology sphere of dimension $r-1$. In fact, this follows from a more general result proved in [4].

In order to make our discussion more precise, we first need to set up some notation and terminology. Indeed, let L denote $\mathbb{Z}$ or $\mathbb{Z}/p$, where p is a prime number. Roughly speaking, a *cohomology r-manifold over L* is a locally compact Hausdorff space which has a local cohomology structure with coefficients in L which is similar to that of the Euclidean r-space. To be precise, all homology groups in this paper are Borel-Moore homology with compact supports

Received March 21, 2019; Accepted July 26, 2019.
2010 *Mathematics Subject Classification.* 20F65, 20F28, 57S25.
Key words and phrases. Higman-Thompson groups, finitely presented infinite simple groups, finite abelian groups, cohomology manifolds, Zimmer program.

and coefficients in a sheaf $\mathcal{A}$ of modules over L. The homology groups of X are denoted by $H_*^c(X;\mathcal{A})$. On the other hand, all cohomology groups in the paper are the Alexander-Spanier cohomology with compact supports and coefficients in $\mathcal{A}$. The cohomology groups of X are denoted by $H_c^*(X;\mathcal{A})$. If the sheaf $\mathcal{A}$ is the constant sheaf $X \times L$, then it will be denoted simply by L. Recall that the cohomology dimension of X with coefficients in L, denoted by $\dim_L X$, is defined by

$$\min\{n \in \mathbb{Z} \,|\, H_c^{n+1}(U;L) = 0 \text{ for all open } U \subset X\}.$$

For each integer $k \geq 0$, let $\mathcal{O}_k(X;L)$ denote the sheaf associated to the presheaf

$$U \mapsto H_k^c(X, X\backslash U; L).$$

A *homology r-manifold X over L* is a locally compact Hausdorff space with $\dim_L X < \infty$ such that $\mathcal{O}_k(X;L) = 0$ for $k \neq r$ and $\mathcal{O}_r(X;L)$ is locally constant with its stalks isomorphic to L. Analogously, a cohomology r-manifold over L can be defined. Note that all topological manifolds of dimension r are a homology and also cohomology r-manifold over L. See [2] and [3] for more details.

With these said, the afore-mentioned result of Bridson and Vogtman in [4] is that if $n \geq 3$, then the unique subgroup $\mathrm{SAut}(F_n)$ of index 2 in the automorphism group $\mathrm{Aut}(F_n)$ of the free group F_n on n generators cannot act effectively by homeomorphisms on any $\mathbb{Z}/2$-acyclic homology manifold of dimension less than n and on any generalized $\mathbb{Z}/2$-homology sphere of dimension less than $n-1$. In [11], Ye proved similar results saying that if $n \geq 3$, then $\mathrm{SAut}(F_n)$ cannot act effectively by homeomorphisms on any connected orientable manifold of dimension less than $n-1$ whose Euler characteristic is not equal to zero modulo 6.

Motivated by the results and the techniques in [11] and Remark 5.1 in [4], in this paper we extend the results of Ye to the actions of certain finitely presented groups by homeomorphisms on a cohomology manifold with a non-zero Euler characteristic. One special feature of the finitely presented groups which we are concerned with is that they contain an isomorphic copy of $(\mathbb{Z}/p)^k$ for any prime p and any positive integer k.

With these said, our main result that is a consequence of Theorem 2.3 is:

Theorem 1.1. *Let X be a first-countable connected cohomology r-manifold over the integers $\mathbb{Z}$ such that the Euler characteristic of X is not equal to 0, and let G denote a group that contains an isomorphic copy of $(\mathbb{Z}/p)^n$ with $n > r$ for any prime p. Then G cannot act effectively on X by homeomorphisms.*

We note that by using the arguments in this paper, Theorem 1.1 can be also derived from [11, Theorem 2.6] together with the fact, mentioned at the beginning of Section 2, that the Euler characteristic does not depend on the coefficient rings used (see Section 2 for more details).

As concrete examples satisfying the condition of Theorem 1.1, there are Higman-Thompson groups in [6] that do not admit any simple description, though. To be more precise, for any $n \geq 2$ and $m \geq 1$ the Higman-Thompson groups $G_{n,m}$ are finitely presented infinite groups that are simple if n is even, and have a simple subgroup $G^+_{n,m}$ of index 2 if n is odd ([6, Theorem 4.6, Theorem 5.4]). One important property of $G^+_{n,m}$ that plays a crucial role in this paper is that $G^+_{n,m}$ contains an isomorphic copy of every countable locally finite group ([6, Theorem 6.6] and [4, Remark 5.1]). Here, a locally finite group means a group for which every finitely generated subgroup is finite. In particular, among many other examples of a locally finite group, every finite group is locally finite. As a consequence, $G^+_{n,m}$ contains an isomorphic copy of every finite group. We remark that infinitely many more examples satisfying the condition of Theorem 1.1 can be found in [1, Proposition 1.4] and [9].

As an immediate consequence of Theorem 1.1, we have the following:

Corollary 1.2. *Let X be a first-countable connected cohomology r-manifold over the integers $\mathbb{Z}$ such that the Euler characteristic of X is not equal to 0, and let $G_{n,m}$ denote the Higman-Thompson group for any $n \geq 2$ and $m \geq 1$. Then $G_{n,m}$ cannot act effectively on X by homeomorphisms.*

By Remark 5.1 in [4] and the results of Mann and Su in [8], it has already been shown that $G_{n,m}$ cannot act effectively by homeomorphisms on any *compact* manifold and on any finite-dimensional $\mathbb{Z}/p$-acyclic homology manifold over $\mathbb{Z}/p$ for any prime p. Clearly, Theorem 1.1 significantly extends these results as well as those in [1], since at any rate the Euler characteristic of any $\mathbb{Z}/p$-acyclic homology manifold over $\mathbb{Z}/p$ is equal to one, so definitely not equal to zero.

In Section 2, we give a proof of Theorem 1.1. To do so, we first prove some general facts necessary for the proof of Theorem 1.1. In particular, we discuss certain motivation of this paper related to the well-known Zimmer program right after Theorem 2.4.

2. Proof of Theorem 1.1

The aim of this section is to give a proof of Theorem 1.1.

To do so, we first need to recall that the Euler characteristic of a cohomology manifold does not depend on the choice of coefficients in a field $\mathbb{F}$ (see [7, Corollary 5.12]). In other words, we have

$$\chi(X) = \chi(X; \mathbb{F})$$

for any cohomology manifold X. It should be remarked that this observation has not been clearly pointed out in the paper [11] and that it actually plays a critical role in dealing with finite abelian groups of rank at least two, as in the proofs of Theorems 2.3 and 2.4.

Now, we begin with the following lemma which seems to be interesting in its own right.

Lemma 2.1. *Let X be a first-countable connected cohomology r-manifold over the integers $\mathbb{Z}$, and let p and q denote distinct primes. Let $G = (\mathbb{Z}/p)^n \times (\mathbb{Z}/q)^m$ act effectively on X by homeomorphisms for positive integers n, m. Then we have*

$$\chi(X) = \sum_{i=0}^{n} \sum_{j=0}^{m} p^i q^{m-j} c_{ij} = \sum_{i=0}^{m} \sum_{j=0}^{n} q^i p^{n-j} d_{ij},$$

where c_{ij} and d_{ij} are some integers.

Proof. For the proof, we use the mathematical induction on n. To do so, let us assume that $n = 1$. Then $G = (\mathbb{Z}/p) \times (\mathbb{Z}/q)^m$ acts effectively on X by homeomorphisms for any $m \in \mathbb{N}$. Choose an element $\alpha \in G$ such that the order of α is p. Let $F := \mathrm{Fix}(\alpha)$. It follows from [2, Theorem 4.4] and [11, Lemma 2.4] that we have

$$\chi(X) = \chi(X - F) + \chi(F). \tag{2.1}$$

Note that $\langle\alpha\rangle$ acts freely on $X - F$ and that $G/\langle\alpha\rangle \cong (\mathbb{Z}/q)^m$ acts on F and $(X - F)/\langle\alpha\rangle$. Thus, it follows from [2, Theorem 3.2] and [11, Lemma 2.4] that

$$\chi(X - F) = p\chi((X - F)/\langle\alpha\rangle).$$

By (2.1), we have

$$\chi(X) = \chi(X - F) + \chi(F) = p\chi((X - F)/\langle\alpha\rangle) + \chi(F). \tag{2.2}$$

On the other hand, by [11, Theorem 2.5] we have

$$\chi((X - F)/\langle\alpha\rangle) = \sum_{j=0}^{m} q^{m-j} a_j, \quad \chi(F) = \sum_{j=0}^{m} q^{m-j} b_j. \tag{2.3}$$

Hence, it follows from (2.2) and (2.3) that

$$\begin{aligned} \chi(X) &= p\chi((X - F)/\langle\alpha\rangle) + \chi(F) \\ &= p \sum_{j=0}^{m} q^{m-j} a_j + \sum_{j=0}^{m} q^{m-j} b_j \\ &= \sum_{i=0}^{1} \sum_{j=0}^{m} p^i q^{m-j} c_{ij}, \quad c_{ij} \in \mathbb{Z}, \end{aligned}$$

which completes the proof for the case $n = 1$.

Next, we assume that the statement of the lemma holds for any $n \geq 2$. So let $G := (\mathbb{Z}/p)^{n+1} \times (\mathbb{Z}/q)^m$ act effectively on X by homeomorphisms for any $m \in \mathbb{N}$. As before, choose an element $\alpha \in G$ such that the order of α is p. Let $F := \mathrm{Fix}(\alpha)$. Note that $\langle\alpha\rangle$ acts freely on $X - F$ and that $G/\langle\alpha\rangle \cong (\mathbb{Z}/p)^n \times (\mathbb{Z}/q)^m$ acts on F and $(X - F)/\langle\alpha\rangle$. Thus, we have

$$\chi(X - F) = p\chi((X - F)/\langle\alpha\rangle).$$

On the other hand, by the induction hypothesis we have

$$\chi((X-F)/\langle\alpha\rangle)=\sum_{i=0}^{n}\sum_{j=0}^{m}p^iq^{m-j}a_{ij},\quad \chi(F)=\sum_{i=0}^{n}\sum_{j=0}^{m}p^iq^{m-j}b_{ij}. \tag{2.4}$$

Hence, it follows from (2.2) and (2.4) that

$$\begin{aligned}\chi(X)&=p\chi((X-F)/\langle\alpha\rangle)+\chi(F)\\&=p\sum_{i=0}^{n}\sum_{j=0}^{m}p^iq^{m-j}a_{ij}+\sum_{i=0}^{n}\sum_{j=0}^{m}p^iq^{m-j}b_{ij}\\&=\sum_{i=0}^{n+1}\sum_{j=0}^{m}p^iq^{m-j}c_{ij},\quad c_{ij}\in\mathbb{Z},\end{aligned}$$

which completes the proof for the case $n+1$.

Similarly, it can be shown that we have

$$\chi(X)=\sum_{i=0}^{m}\sum_{j=0}^{n}q^ip^{n-j}d_{ij},\quad d_{ij}\in\mathbb{Z}.$$

This completes the proof of Lemma 2.1. □

More generally, it is now immediate to obtain the following lemma whose proof will be left to a reader.

Lemma 2.2. *Let X be a first-countable connected cohomology r-manifold over the integers $\mathbb{Z}$, and let $p_1,p_2,\dots,p_k$ denote distinct primes. Let*

$$G=(\mathbb{Z}/p_1)^{n_1}\times\cdots\times(\mathbb{Z}/p_k)^{n_k}$$

act effectively on X by homeomorphisms for positive integers $n_1,n_2,\dots,n_k$. Then we have

$$\begin{aligned}\chi(X)&=\sum_{i_1=0}^{n_1}\sum_{i_2=0}^{n_2}\cdots\sum_{i_k=0}^{n_k}p_1^{i_1}p_2^{i_2}\cdots p_{k-1}^{i_{k-1}}p_k^{n_k-i_k}c^k_{i_1i_2\cdots i_k}\\&=\sum_{i_1=0}^{n_1}\sum_{i_2=0}^{n_2}\cdots\sum_{i_k=0}^{n_k}p_1^{i_1}p_2^{i_2}\cdots p_{k-1}^{n_{k-1}-i_{k-1}}p_k^{i_k}c^{k-1}_{i_1i_2\cdots i_k}\\&=\cdots\\&=\sum_{i_1=0}^{n_1}\sum_{i_2=0}^{n_2}\cdots\sum_{i_k=0}^{n_k}p_1^{n_1-i_1}p_2^{i_2}\cdots p_{k-1}^{i_{k-1}}p_k^{n_k}c^1_{i_1i_2\cdots i_k},\end{aligned}$$

where $c^j_{i_1i_2\cdots i_k}$ are some integers.

For a group G and a prime integer p, recall that the *p-rank of G*, denoted by $\mathrm{rk}_p(G)$, is defined to be

$$\mathrm{rk}_p(G)=\sup\{k\in\mathbb{Z}\mid(\mathbb{Z}/p)^k\text{ is embedded into }G\}.$$

Note that it is possible to have $\mathrm{rk}_p(G) = \infty$. In this paper, we denote by $\mathrm{Homeo}(X)$ the group of self-homeomorphisms of a cohomology manifold X over $L = \mathbb{Z}$ or $\mathbb{Z}/p$. As usual, in the next theorem we use the convention that $p^n = 1$ whenever $n < 0$.

Theorem 2.3. *Let X be a first-countable connected cohomology r-manifold over the integers $\mathbb{Z}$, and let $p_1, p_2, \ldots, p_k$ denote distinct primes. Let*

$$G = (\mathbb{Z}/p_1)^{n_1} \times \cdots \times (\mathbb{Z}/p_k)^{n_k}$$

act effectively on X by homeomorphisms for positive integers $n_1, n_2, \ldots, n_k$ such that for each $i = 1, 2, \ldots, k$, we have

$$\mathrm{rk}_{p_i}(\mathrm{Homeo}(X)) = n_i.$$

Then the following statements hold.

(1) *If the action of G is free, then we have*

$$p_1^{n_1} p_2^{n_2} \cdots p_k^{n_k} | \chi(X).$$

(2) *If the action of G is not free, then we have*

$$\begin{cases} p_1^{n_1-[\frac{r}{2}]} p_2^{n_2-[\frac{r}{2}]} \cdots p_k^{n_k-[\frac{r}{2}]} | \chi(X), & \textit{if } p_1, p_2, \ldots, p_k \textit{ are all odd,} \\ 2^{n_1-r} p_2^{n_2-[\frac{r}{2}]} \cdots p_k^{n_k-[\frac{r}{2}]} | \chi(X), & \textit{if } p_1 = 2, p_2, p_3, \ldots, p_k \textit{ are all odd.} \end{cases}$$

Proof. (1) If the action G is free, then it follows from [11, Theorem 2.5] or Lemma 2.1 that for each $i = 1, 2, \ldots, k$ we have p^{n_i} divides $\chi(X)$. Since all of $p_1, p_2, \ldots, p_k$ are mutually prime, clearly $p_1^{n_1} p_2^{n_2} \cdots p_k^{n_k}$ divides $\chi(X)$.

(2) Now, assume that the action of G is not free. Let $H := (\mathbb{Z}/p_k)^{n_k}$ as a subgroup of G. Then H acts effectively on X by homeomorphisms. Let

$$X_i := \{x \in X \mid |H_x| = p_k^i\},$$

where H_x denotes the stabilizer of H at x. Then it follows from [11, Lemma 2.2] that for each $x \in X_i$ we have

$$\mathrm{rk}_{p_k}(H_x) = i \le \begin{cases} \frac{r}{2}, & \text{if } p_k \neq 2, \\ r, & \text{if } p_k = 2. \end{cases}$$

Thus for any i $(1 \le i \le n_k)$ it is easy to obtain

$$p_k^{n_k - i} \ge \begin{cases} p_k^{n_k - \frac{r}{2}}, & \text{if } p_k \neq 2, \\ p_k^{n_k - r}, & \text{if } p_k = 2. \end{cases}$$

It then follows from Lemma 2.2 (or a direct argument as in the proof of Lemma 2.1 or 2.2) that

$$\begin{cases} p_k^{n_k - \frac{r}{2}} | \chi(X), & \text{if } p_k \neq 2, \\ p_k^{n_k - r} | \chi(X), & \text{if } p_k = 2. \end{cases}$$

Similarly, by applying the same arguments to p_j $(1 \le j \le k-1)$, we can show

$$\begin{cases} p_j^{n_j - \frac{r}{2}} | \chi(X), & \text{if } p_j \neq 2, \\ p_j^{n_j - r} | \chi(X), & \text{if } p_j = 2. \end{cases}$$

Since all of $p_1, p_2, \ldots, p_k$ are mutually prime, it is immediate to obtain the desired divisibilities of the Euler characteristic of X. This completes the proof of Theorem 2.3. □

Theorem 2.4. *Let X be a first-countable connected cohomology r-manifold over the integers $\mathbb{Z}$, and let $p_1, p_2, \ldots, p_k$ denote distinct odd primes. Then the following statements hold.*

(1) *Assume that $n_1, n_2, \ldots, n_k \ge [\frac{r}{2}]$, but not all of them are equal to $[\frac{r}{2}]$ and that the Euler characteristic $\chi(X)$ of X is not equal to zero modulo*
$$p_1^{n_1-[\frac{r}{2}]} p_2^{n_2-[\frac{r}{2}]} \cdots p_k^{n_k-[\frac{r}{2}]}.$$
Then
$$G := (\mathbb{Z}/p_1)^{n_1} \times \cdots \times (\mathbb{Z}/p_k)^{n_k}$$
cannot act effectively on X by homeomorphisms.

(2) *Assume that $n_0 \ge r$ and $n_1, n_2, \ldots, n_k \ge [\frac{r}{2}]$, but $n_0 \neq r$ or not all of them are equal to $[\frac{r}{2}]$ and that the Euler characteristic $\chi(X)$ of X is not equal to zero modulo*
$$2^{n_0-r} p_1^{n_1-[\frac{r}{2}]} p_2^{n_2-[\frac{r}{2}]} \cdots p_k^{n_k-[\frac{r}{2}]}.$$
Then
$$G := (\mathbb{Z}/2)^{n_0} \times (\mathbb{Z}/p_1)^{n_1} \times \cdots \times (\mathbb{Z}/p_k)^{n_k}$$
cannot act effectively on X by homeomorphisms.

Proof. Suppose that G acts effectively on X by homeomorphisms. Then it follows from Theorem 2.3 that we have

$$\begin{cases} p_1^{n_1-[\frac{r}{2}]} p_2^{n_2-[\frac{r}{2}]} \cdots p_k^{n_k-[\frac{r}{2}]} | \chi(X), & \text{if } p_1, p_2, \ldots, p_k \text{ are all odd}, \\ 2^{n_1-r} p_2^{n_2-[\frac{r}{2}]} \cdots p_k^{n_k-[\frac{r}{2}]} | \chi(X), & \text{if } p_1 = 2, p_2, p_3, \ldots, p_k \text{ are all odd}. \end{cases}$$

This is clearly a contradiction to the assumptions made in (1) and (2) of Theorem 1.1. This completes the proof. □

Theorem 2.4 generalizes [11, Theorem 2.6] that plays an important role in the proof of [11, Theorem 1.2] for the actions of $\mathrm{SAut}(F_n)$ by homeomorphisms on a cohomology manifold with certain non-zero Euler characteristic. Actually, Ye makes use of the rigidity property of a finite abelian group $(\mathbb{Z}/3)^m$ embedded in $\mathrm{SAut}(F_n)$ as in [4].

It is clear that for any compact connected manifold X of dimension less than $n-1$ with a non-zero Euler characteristic, there exists a prime p for which the Euler characteristic is not equal to zero modulo p. Hence, if $SL(n, \mathbb{Z})$ contains an isomorphic copy of $(\mathbb{Z}/p)^m$ for such a prime p, then it follows from Theorem 2.4 that the action of $SL(n, \mathbb{Z})$ by homeomorphisms on X should be trivial.

In view of this discussion, it would be interesting to ask if we can find some finite abelian groups of the form $(\mathbb{Z}/p_1)^{n_1} \times \cdots \times (\mathbb{Z}/p_k)^{n_k}$ embedded in $\mathrm{SAut}(F_n)$. As mentioned in Section 1, it is well-known that the Higman-Thompson groups $G_{n,m}$ contain an isomorphic copy of every finite group.

Finally, we are ready to give a proof of Theorem 1.1.

Proof of Theorem 1.1. Since our group G contain an isomorphic copy of $(\mathbb{Z}/p)^n$ with $n > r$ for any prime p, it follows from Theorem 2.4 that if X admits an effective action of G, the Euler characteristic $\chi(X)$ of X should be equal to zero modulo p for any prime p. Hence $\chi(X)$ should be equal to zero, which is a contradiction. This completes the proof. □

Acknowledgements: The author is very grateful to the anonymous reader for providing the valuable comments on this paper. This study was supported by research fund from Chosun University, 2019. This research was also supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (NRF-2016R1D1A1B03930639).

References

[1] G. Arzhantseva, M. Bridson, T. Januszkiewicz, I. Leary, A. Minasyan, and J. Świątkowski, *Infinite groups with fixed point properties*, Geom. Topol. **13** (2009), no. 3, 1229–1263. https://doi.org/10.2140/gt.2009.13.1229

[2] A. Borel, *Seminar on Transformation Groups*, With contributions by G. Bredon, E. E. Floyd, D. Montgomery, R. Palais. Annals of Mathematics Studies, No. 46, Princeton University Press, Princeton, NJ, 1960.

[3] G. E. Bredon, *Sheaf Theory*, second edition, Graduate Texts in Mathematics, **170**, Springer-Verlag, New York, 1997. https://doi.org/10.1007/978-1-4612-0647-7

[4] M. R. Bridson and K. Vogtmann, *Actions of automorphism groups of free groups on homology spheres and acyclic manifolds*, Comment. Math. Helv. **86** (2011), no. 1, 73–90. https://doi.org/10.4171/CMH/218

[5] D. Fisher, *Groups acting on manifolds: around the Zimmer program*, in Geometry, rigidity, and group actions, 72–157, Chicago Lectures in Math, Univ. Chicago Press, Chicago, IL, 2011. https://doi.org/10.7208/chicago/9780226237909.001.0001

[6] G. Higman, *Finitely presented infinite simple groups*, Department of Pure Mathematics, Department of Mathematics, I.A.S. Australian National University, Canberra, 1974.

[7] K. Kawakubo, *The Theory of Transformation Groups*, translated from the 1987 Japanese edition, The Clarendon Press, Oxford University Press, New York, 1991.

[8] L. N. Mann and J. C. Su, *Actions of elementary p-groups on manifolds*, Trans. Amer. Math. Soc. **106** (1963), 115–126. https://doi.org/10.2307/1993717

[9] C. E. Röver, *Constructing finitely presented simple groups that contain Grigorchuk groups*, J. Algebra **220** (1999), no. 1, 284–313. https://doi.org/10.1006/jabr.1999.7898

[10] D. Witte, *Arithmetic groups of higher $\mathbb{Q}$-rank cannot act on 1-manifolds*, Proc. Amer. Math. Soc. **122** (1994), no. 2, 333–340. https://doi.org/10.2307/2161021

[11] S. Ye, *Euler characteristics and actions of automorphism groups of free groups*, Algebr. Geom. Topol. **18** (2018), no. 2, 1195–1204. https://doi.org/10.2140/agt.2018.18.1195

Jin Hong Kim
Department of Mathematics Education
Chosun University
Gwangju 61452, Korea
Email address: jinhkim11@gmail.com