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ON RECTIFYING-TYPE CURVES IN A MYLLER CONFIGURATION

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Abstract. We consider special curves (rectifying-type curves) in the simplest Myller configuration and study their properties, in order to compare these properties in both cases, Myller and Euclidean settings.

1. Introduction

In the differential geometry of curves in the Euclidean space $\mathbf{E}_3$ one introduces, along a curve $\mathcal{C}$, standard vector fields, as tangent $\overrightarrow{t}$, principal normal $\overrightarrow{n}$ and binormal $\overrightarrow{b}$, as well as osculating, normal or rectifying planes, generated respectively by $(\overrightarrow{t}, \overrightarrow{n})$, $(\overrightarrow{n}, \overrightarrow{b})$ and $(\overrightarrow{t}, \overrightarrow{b})$.

More generally, one considers a *versor field* (i.e., a unit vector field) $(\mathcal{C}, \overline{\xi})$ and a plane field $(\mathcal{C}, \pi)$. A pair $\{(\mathcal{C}, \overline{\xi}), (\mathcal{C}, \pi)\}$ such that $\overline{\xi} \in \pi$ is called a *Myller configuration* in $\mathbf{E}_3$ and is denoted by $\mathcal{M}(\mathcal{C}, \overline{\xi}, \pi)$. If, moreover, the planes π are tangent to $\mathcal{C}$, then we have a *tangent Myller configuration* $\mathcal{M}_t(\mathcal{C}, \overline{\xi}, \pi)$.

In [6] a systematic presentation of the geometry of Myller configurations $\mathcal{M}(\mathcal{C}, \overline{\xi}, \pi)$ and $\mathcal{M}_t(\mathcal{C}, \overline{\xi}, \pi)$ is given.

If $\mathcal{C}$ is a curve on a surface $\mathcal{S} \subset \mathbf{E}_3$, s is the natural parameter of $\mathcal{C}$, $\overline{\xi}(s)$ are the tangent vector field to $\mathcal{S}$ along $\mathcal{C}$, $\pi(s)$ are the tangent plane field to $\mathcal{S}$ along $\mathcal{C}$, then $\mathcal{M}_t(\mathcal{C}, \overline{\xi}, \pi)$ is the tangent Myller configuration intrinsic associated to the geometric objects $\mathcal{S}$, $\mathcal{C}$, $\overline{\xi}$.

The geometry of the vector field $(\mathcal{C}, \overline{\xi})$ on a surface $\mathcal{S}$ is the geometry of the associated Myller configurations $\mathcal{M}_t(\mathcal{C}, \overline{\xi}, \pi)$. The geometric theory of $\mathcal{M}_t(\mathcal{C}, \overline{\xi}, \pi)$ represents a particular case of the general Myller configuration $\mathcal{M}(\mathcal{C}, \overline{\xi}, \pi)$. In the case when $\mathcal{M}_t(\mathcal{C}, \overline{\xi}, \pi)$ is the associated Myller configuration to a curve $\mathcal{C}$ on a surface $\mathcal{S}$ one obtains the classical theory of curves on surfaces.

We cite from Miron's book [6] (Preface, written in 1966 by O. Mayer, and Introduction):

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"Initiator of a modern education of Mathematics at the University of Iassy, founder of the Geometry School, which is still flourishing today, in the third generation, Alexandru Myller was also a hardworking researcher, well known inside the country and also abroad due his papers concerning Integral Equations and Differential Geometry. Some of Alexandru Myller's discoveries have penetrated fruitfully the impetuous torrent of ideas, which have changed the Science of Geometry during the first decades of the century. Among others, it is the case of "Myller configurations".

Academician Alexandru Myller studied in 1922 the notion of parallelism of $\left(\mathcal{C},\overline{\xi}\right)$ in the plane field $(\mathcal{C},\pi)$ obtaining an interesting generalization of the famous parallelism of Levi-Civita on the curved surfaces. These investigations have been continued by Octav Mayer who introduced new fundamental invariants for $\mathcal{M}(\mathcal{C},\overline{\xi},\pi)$ and $\mathcal{M}_t(\mathcal{C},\overline{\xi},\pi)$. The importance of these studies was underlined by Levi-Civita in Addendum to his book *Lezioni di calcolo differentiale assoluto,* 1925."

The notion of a Myller configuration was extended to Riemannian geometry, symplectic geometry, in the geometry of versor fields in the Euclidean space with applications in hydromechanics, in Minkowski spaces, in Finsler, Lagrange and Hamilton spaces.

2. Versor fields in $\mathbf{E_3}$. Frame of Frenet-type

One considers a *versor field* $\left(\mathcal{C},\overline{\xi}\right)$ in the Euclidean space $\mathbf{E}_3$. An *invariant frame of Frenet-type* is introduced and the moving equations of this frame and the invariants $K_1(s)$, $K_2(s)$ are presented in [6].

In $\mathbf{E}_3$, a versor field $\left(\mathcal{C},\overline{\xi}\right)$ can be analytical represented in an orthonormal frame $\mathcal{R}=(O;\overrightarrow{i_1},\overrightarrow{i_2},\overrightarrow{i_3})$ by

$$\overline{r}=\overline{r}(s),\ \overline{\xi}=\overline{\xi}(s),\ s\in I=(s_1,s_2),$$

where s is the arclength on the curve $\mathcal{C}$,

$$\overline{r}(s)=x(s)\overrightarrow{i_1}+y(s)\overrightarrow{i_2}+z(s)\overrightarrow{i_3}=\overrightarrow{OP}(s),$$

$$\overline{\xi}(s)=\xi_1(s)\overrightarrow{i_1}+\xi_2(s)\overrightarrow{i_2}+\xi_3(s)\overrightarrow{i_3}=\overrightarrow{PQ},$$

with $||\overline{\xi}(s)||^2=\langle\overline{\xi}(s),\overline{\xi}(s)\rangle=1$.

$K_1(s)=||\frac{d\overline{\xi}}{ds}||$ is an invariant of the field $\left(\mathcal{C},\overline{\xi}\right)$.

Denote $\overline{\xi}_1(s)=\overline{\xi}(s)$ and by $\overline{\xi}_2(s)$ the versor of $\frac{d\overline{\xi}_1(s)}{ds}$, i.e.,

$$\frac{d\overline{\xi}_1(s)}{ds}=K_1(s)\overline{\xi}_2(s).$$

Obviously, $\overline{\xi}_2(s)$ is orthogonal to $\overline{\xi}_1(s)$ and exists if $\frac{d\overline{\xi}_1(s)}{ds}$ is non-zero. One defines $\overline{\xi}_3(s)=\overline{\xi}_1(s)\times\overline{\xi}_2(s)$.

Then the frame $\mathcal{R}_{\mathcal{F}}(P(s); \overline{\xi}_1(s), \overline{\xi}_2(s), \overline{\xi}_3(s))$ is orthonormal, positively oriented. $\mathcal{R}_{\mathcal{F}}$ is called *the Frenet frame of the versor field* $\left(\mathcal{C}, \overline{\xi}\right)$ (invariant *frame of Frenet-type*).

The *moving equations* of $\mathcal{R}_{\mathcal{F}}$ are

$$\frac{d\overline{r}(s)}{ds} = a_1(s)\overline{\xi}_1(s) + a_2(s)\overline{\xi}_2(s) + a_3(s)\overline{\xi}_3(s),$$

with $a_1^2(s) + a_2^2(s) + a_3^2(s) = 1$ and

$$\begin{aligned} \frac{d\overline{\xi}_1(s)}{ds} &= K_1(s)\overline{\xi}_2(s), \\ \frac{d\overline{\xi}_2(s)}{ds} &= -K_1(s)\overline{\xi}_1(s) + K_2(s)\overline{\xi}_3(s), \\ \frac{d\overline{\xi}_3(s)}{ds} &= -K_2(s)\overline{\xi}_2(s), \end{aligned}$$

where $K_1(s) > 0$.

The functions $K_1(s)$, $K_2(s)$, $a_1(s)$, $a_2(s)$, $a_3(s)$, $s \in I$ are invariants of the versor field $\left(\mathcal{C}, \overline{\xi}\right)$. $K_1(s)$ is the *curvature* of $\left(\mathcal{C}, \overline{\xi}\right)$ (also called K_1-*curvature*). $K_2(s)$ is the *torsion* of $\left(\mathcal{C}, \overline{\xi}\right)$ (also called K_2-*torsion*). $K_1(s)$, $K_2(s)$ have the same geometrical interpretation as the curvature and torsion of a curve in $\mathbf{E}_3$.

Obviously, if $a_1(s) = 1$, $a_2(s) = 0$, $a_3(s) = 0$, one obtains the Frenet equations of a curve in $\mathbf{E}_3$.

The fundamental theorem for the versor field $\left(\mathcal{C}, \overline{\xi}\right)$ is the following:

Theorem 1 ([6])**.** *If the functions $K_1(s)$, $K_2(s)$, $a_1(s)$, $a_2(s)$, $a_3(s)$, $(a_1^2(s) + a_2^2(s) + a_3^2(s) = 1)$ of class C^∞ are a priori given, $s \in [a, b]$, then there exists a curve $\mathcal{C} : [a, b] \to \mathbf{E}_3$ parametrized by arclength s and a versor field $\overline{\xi}(s)$, $s \in [a, b]$, whose the curvature, torsion and the functions $a_i(s)$ are $K_1(s)$, $K_2(s)$ and $a_i(s)$, $i = 1, 2, 3$. Any two such versor fields $\left(\mathcal{C}, \overline{\xi}\right)$ differ by a proper Euclidean motion.*

We list the following geometric properties [6]:

1. The versor field $\left(\mathcal{C}, \overline{\xi}\right)$ determines a ruled surface $\mathcal{S}\left(\mathcal{C}, \overline{\xi}\right)$.
2. The surface $\mathcal{S}\left(\mathcal{C}, \overline{\xi}\right)$ is with director plane if and only if $K_2(s) = 0$.
3. The surface $\mathcal{S}\left(\mathcal{C}, \overline{\xi}\right)$ is developing if and only if the invariant $a_3(s)$ vanishes.

If the surface $\mathcal{S}\left(\mathcal{C}, \overline{\xi}\right)$ is a cone, the versor field $\left(\mathcal{C}, \overline{\xi}\right)$ is called *concurrent.*

3. Curves in the Euclidean space. Rectifying curves

Consider a unit-speed curve $x : I \to \mathbf{E}_3$ and the (classical) Frenet frame $\{\overrightarrow{t}, \overrightarrow{n}, \overrightarrow{b}\}$, where $\overrightarrow{t}$ is the *tangent vector field*, $\overrightarrow{n}$ is the *principal normal vector field*, $\overrightarrow{b}$ is the *binormal vector field.* At each point of the curve, the planes spanned by $(\overrightarrow{t}, \overrightarrow{n})$, $(\overrightarrow{t}, \overrightarrow{b})$ and $(\overrightarrow{n}, \overrightarrow{b})$ are known as the *osculating plane*, the *rectifying plane* and the *normal plane*, respectively.

From elementary differential geometry it is well known that a curve in $\mathbf{E}_3$ lies in a plane if its position vector lies in its osculating plane at each point and lies on a sphere if its position vector lies in its normal plane at each point. In [1], B.-Y. Chen asked the following simple geometric question:

Question. When does the position vector of a space curve $x : I \to \mathbf{E}_3$ always lie in its rectifying plane?

Such a curve is called a *rectifying curve* and its position vector x satisfies

$$x(s) = \lambda(s)\overrightarrow{t}(s) + \mu(s)\overrightarrow{b}(s)$$

for some functions λ and μ.

In [1] the author derived many fundamental properties of rectifying curves. Characterizations of rectifying curves are given and, moreover, classification of such curves is provided.

By applying a result from [1], one concludes that, up to rigid motion, rectifying curves are characterized, in terms of mechanics, as those curves that are in equilibrium under a force field $F = c\overrightarrow{t} - \tau\overrightarrow{n}$, for a constant $c \neq 0$, when τ is the torsion of the curve x. Other results were obtained in [2] and [3].

Furthermore, B.-Y. Chen and F. Dillen established in [4] a simple link between rectifying curves and the notion of centrodes in mechanics. They also showed that rectifying curves are indeed the extremal curves which satisfy the equality case of a general inequality.

In [5], S. Deshmukh, B.-Y. Chen and S. H. Al Shammari studied rectifying curves via the dilation of unit speed curves on the unit sphere S^2 in the Euclidean space $\mathbf{E}_3$. A main result of [4] is improved.

As we already mentioned in the abstract, in this short note we consider special curves (rectifying-type curves) in the most simple Myller configuration and study their properties, in order to compare these properties in both cases, Myller and Euclidean settings.

4. Rectifying curves in a Frenet-type frame

We consider the curve $\overline{r}(s)$ and a frame of Frenet-type $\mathcal{R}_{\mathcal{F}} = \{P(s); \overline{\xi}_1(s), \overline{\xi}_2(s), \overline{\xi}_3(s)\}$.

Denote by:

$$\frac{d\overline{r}}{ds} = \dot{\overline{r}}(s)$$

and

$$\frac{d\overline{\xi}_1(s)}{ds} = \dot{\overline{\xi}}_1(s),$$
$$\frac{d\overline{\xi}_2(s)}{ds} = \dot{\overline{\xi}}_2(s),$$
$$\frac{d\overline{\xi}_3(s)}{ds} = \dot{\overline{\xi}}_3(s).$$

We generalize the definition of a rectifying curve by:

Definition. $\overline{r}$ is a *rectifying-type curve* (or simply *rectifying curve*) in the Frenet-type frame $\mathcal{R}_{\mathcal{F}}$ if

$$\overline{r}(s) = \lambda(s)\overline{\xi}_1(s) + \mu(s)\overline{\xi}_3(s), \tag{1}$$

where λ, μ are functions.

Remark. From relation (1) we have, by using the moving equations of $\mathcal{R}_{\mathcal{F}}$ (from Section 2):

$$\begin{aligned} \dot{\overline{r}}(s) &= \dot{\lambda}(s)\overline{\xi}_1(s) + \lambda(s)\dot{\overline{\xi}}_1(s) + \dot{\mu}(s)\overline{\xi}_3(s) + \mu(s)\dot{\overline{\xi}}_3(s) \\ &= \dot{\lambda}(s)\overline{\xi}_1(s) + \lambda(s)K_1(s)\overline{\xi}_2(s) + \dot{\mu}(s)\overline{\xi}_3(s) + \mu(s)(-K_2(s)\overline{\xi}_2(s)) \\ &= \dot{\lambda}(s)\overline{\xi}_1(s) + [\lambda(s)K_1(s) - \mu(s)K_2(s)]\overline{\xi}_2(s) + \dot{\mu}(s)\overline{\xi}_3(s). \end{aligned} \tag{2}$$

From relation (2) and the expression of $\dot{\overline{r}}(s)$ in a Frenet-type frame, we obtain the following formulae:

$$\begin{aligned} \dot{\lambda}(s) &= a_1(s), \\ \lambda(s)K_1(s) - \mu(s)K_2(s) &= a_2(s), \\ \dot{\mu}(s) &= a_3(s), \end{aligned} \tag{3}$$

with $a_1^2(s) + a_2^2(s) + a_3^2(s) = 1$.

We give the following characterization of the rectifying-type curves:

Theorem 2. *Let $\overline{r}(s) : I \to \mathbf{E}_3$ be a curve in $\mathbf{E}_3$ expressed in the Frenet-type frame $\mathcal{R}_{\mathcal{F}}$ by:*

$$\frac{d\overline{r}}{ds}(s) = a_1(s)\overline{\xi}_1(s) + a_2(s)\overline{\xi}_2(s) + a_3(s)\overline{\xi}_3(s),$$

with $K_1(s) > 0$, such that one of the following items holds:
i) $\frac{d}{ds}\langle\overline{r}(s), \overline{\xi}_1(s)\rangle = a_1(s)$,
ii) *For* $K_2(s) \neq 0, \frac{d}{ds}\langle\overline{r}(s), \overline{\xi}_3(s)\rangle = a_3(s)$.
Then $\overline{r}(s)$ is a rectifying-type curve.
Conversely, if $\overline{r}(s)$ is a rectifying-type curve, then i) *and* ii) *hold.*

Proof. If $\overline{r}(s)$ is a rectifying-type curve, then i) and ii) hold, according to the previous remark.

To prove the direct implication, assume that i) holds. Then

$$\langle\overline{r}(s), \overline{\xi}_1(s)\rangle = \lambda(s).$$

By taking the derivatives with respect to s we find

$$\langle\dot{\overline{r}}(s), \overline{\xi}_1(s)\rangle + \langle\overline{r}(s), \dot{\overline{\xi}}_1(s)\rangle = \dot{\lambda}(s).$$

This implies

$$\langle a_1(s)\overline{\xi}_1(s) + a_2(s)\overline{\xi}_2(s) + a_3(s)\overline{\xi}_3(s), \overline{\xi}_1(s)\rangle + \langle\overline{r}(s), K_1(s)\overline{\xi}_2(s)\rangle = a_1(s),$$

which gives

$$a_1(s) + K_1(s)\langle \overline{r}(s), \overline{\xi}_2(s)\rangle = a_1(s) \Rightarrow \langle \overline{r}(s), \overline{\xi}_2(s)\rangle = 0,$$

because $K_1(s) > 0$.

Also ii) implies

$$\langle \dot{\overline{r}}(s), \overline{\xi}_3(s)\rangle + \langle \overline{r}(s), \dot{\overline{\xi}}_3(s)\rangle = \dot{\mu}(s)$$
$$\Rightarrow a_3(s) - K_2(s)\langle \overline{r}(s), \overline{\xi}_2(s)\rangle = a_3(s).$$

But $K_2(s) \neq 0$, so $\langle \overline{r}(s), \overline{\xi}_2(s)\rangle = 0$.

It follows that in both cases i) or ii) implies that $\overline{r}(s)$ is a rectifying-type curve. □

On the other hand, denoting by $\rho(s)$ the distance function, $\rho(s) = ||\overline{r}(s)||$ of a rectifying-type curve $\overline{r}(s)$, we define

$$y(s) = \frac{1}{||\overline{r}(s)||} \cdot \overline{r}(s) = \frac{1}{\rho(s)} \cdot \overline{r}(s)$$

a unit curve.

Then $\rho(s) = \sqrt{\lambda^2(s) + \mu^2(s)}$ and $||y(s)|| = 1$.

It follows that

$$\dot{\rho}(s) = \frac{\lambda(s) \cdot \dot{\lambda}(s) + \mu(s) \cdot \dot{\mu}(s)}{\sqrt{\lambda^2(s) + \mu^2(s)}} = \frac{a_1(s) \cdot \lambda(s) + a_3(s) \cdot \mu(s)}{\sqrt{\lambda^2(s) + \mu^2(s)}}. \tag{4}$$

Also $1 = ||y(s)||^2 = \langle y(s), y(s)\rangle$ implies $\langle y(s), \dot{y}(s)\rangle = 0$.

From $\overline{r}(s) = \rho(s)y(s)$, we obtain

$$\dot{\overline{r}}(s) = \dot{\rho}(s)y(s) + \rho(s)\dot{y}(s)$$

and denoting by $v = ||\dot{y}(s)||$ the speed of the curve $y(s)$ it follows that

$$1 = \dot{\rho}^2(s) + \rho^2(s)v^2,$$

because $||\dot{\overline{r}}(s)||^2 = a_1^2(s) + a_2^2(s) + a_3^2(s) = 1$ from the moving equations.

Then

$$v^2 = \frac{1 - \dot{\rho}^2(s)}{\rho^2(s)}. \tag{5}$$

From relations (4) and (5) we have, if $a_2(s) = 0$:

$$v = \frac{|a_3(s)\lambda(s) - a_1(s)\mu(s)|}{\lambda^2(s) + \mu^2(s)}.$$

Moreover, if the functions $a_1(s)$ and $a_3(s)$ are constant, $a_1(s) = a_1$, $a_3(s) = a_3$, then

$$\lambda(s) = a_1 s + b_1, \quad \mu(s) = a_3 s + b_3.$$

It follows that

$$v = \frac{|a_3(a_1 s + b_1) - a_1(a_3 s + b_3)|}{(a_1 s + b_1)^2 + (a_3 s + b_3)^2}$$

$$= \frac{|a_3 b_1 - a_1 b_3|}{s^2 + 2(a_1 b_1 + a_3 b_3)s + b_1^2 + b_3^2}.$$

We write

$$v = \frac{c}{[s + (a_1 b_1 + a_3 b_3)]^2 + [b_1^2 + b_3^2 - (a_1 b_1 + a_3 b_3)^2]}.$$

By a translation, we may assume $v = \frac{c}{s^2+d^2}$, $d > 0$.
Denote

$$t = \frac{d}{c}\int_0^s v(u)du = \frac{d}{c}\int_0^s \frac{c}{u^2+d^2}du = \frac{d}{c}\cdot\frac{c}{d}\arctan\frac{s}{d} = \arctan\frac{s}{d}.$$

Then $s(t) = d\tan t$ and

$$\begin{aligned}\overline{r}(t) &= \overline{r}(s(t)) = \rho(s(t))\cdot y(s(t)) = \sqrt{s^2(t)+d^2}\cdot y(s(t))\\ &= \sqrt{d^2\tan^2 t + d^2}\cdot y(s(t)) = d\cdot\sqrt{1+\tan^2 t}\cdot y(s(t))\\ &= d\sec t\cdot y(s(t)) = d\cdot\sec t\cdot y(t).\end{aligned}$$

Also, $\dot{y}(t) = \dot{y}(s)\cdot\frac{ds}{dt}$ implies that

$$||\dot{y}(t)|| = ||\dot{y}(s)||\cdot\frac{ds}{dt} = v\cdot\frac{ds}{dt} = v\cdot\frac{1}{\frac{dt}{ds}} = v\cdot(\frac{d}{c}\cdot v)^{-1} = \frac{c}{d}.$$

Theorem 3. *A rectifying-type curve $\overline{r}(s)$ in a Frenet-type frame $\mathcal{R}_{\mathcal{F}}$ with $a_2(s) = 0$, $a_1(s) = a_1$, $a_3(s) = a_3$, a_1, a_3 = constants, is given, up to a parametrization, by*

$$\overline{r}(s) = (d\cdot\sec t)y(t),$$

where d is a positive number and $y = y(t)$ is a constant speed curve in S^2.

Remark. The converse statement holds if $a_2 = a_3 = 0$, then $a_1 = 1$ and the Frenet-type frame $\mathcal{R}_{\mathcal{F}}$ coincides with classical Frenet frame.

In the last part of this section, we will characterize a rectifying-type curve, via its K_1-curvature and K_2-torsion.

$$\begin{aligned}&\frac{d}{ds}(\overline{r}(s) - \lambda(s)\overline{\xi}_1(s) - \mu(s)\overline{\xi}_3(s))\\ &= [a_1(s)\overline{\xi}_1(s) + a_2(s)\overline{\xi}_2(s) + a_3(s)\overline{\xi}_3(s)]\\ &\quad - \dot{\lambda}(s)\overline{\xi}_1(s) - \lambda(s)\dot{\overline{\xi}}_1(s) - \dot{\mu}(s)\overline{\xi}_3(s) - \mu(s)\dot{\overline{\xi}}_3(s)\\ &= a_1(s)\overline{\xi}_1(s) + a_2(s)\overline{\xi}_2(s) + a_3(s)\overline{\xi}_3(s) - a_1(s)\overline{\xi}_1(s)\\ &\quad - \lambda(s)K_1(s)\overline{\xi}_2(s) - a_3(s)\overline{\xi}_3(s) - \mu(s)(-K_2(s)\overline{\xi}_2(s))\\ &= [a_2(s) - \lambda(s)K_1(s) + \mu(s)K_2(s)]\overline{\xi}_2(s) = 0,\end{aligned}$$

according to the relation (3).

The previous theorem gives a very simple characterization of rectifying curves if $a_2(s) = 0$ and $K_2(s) \neq 0$, in terms of the ratio $\frac{K_2(s)}{K_1(s)}$.

Theorem 4. *Let $\overline{r} : I \to \mathbf{E}_3$ be a curve in the Frenet-type frame $\mathcal{R}_{\mathcal{F}}$, $\dot{\overline{r}}(s) = a_1(s)\overline{\xi}_1(s) + a_2(s)\overline{\xi}_2(s) + a_3(s)\overline{\xi}_3(s)$.*

Then $\overline{r}$ is congruent to a rectifying-type curve

$$\overline{r}(s) = \lambda(s)\overline{\xi}_1(s) + \mu(s)\overline{\xi}_3(s)$$

if and only if the K_1-curvature and the K_2-torsion satisfy the relation

$$\lambda(s)K_1(s) - \mu(s)K_2(s) = a_2(s),\ \forall s \in I.$$

Remark. Recall that a curve in $\mathbf{E}_3$ is a generalized helix if and only if the ratio $\frac{\tau}{K}$ (τ=torsion, K=curvature) is a nonzero constant on the curve.

We conclude that the properties of rectifying-type curves in a Myller configuration differ from those of the rectifying curves in the Euclidean space and, based on this remark, we consider that the definitions and studies of some other special curves in Myller configurations would be of real interest in differential geometry.

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