Bull. Korean Math. Soc. **0** (0), No. 0, pp. 1–0
https://doi.org/10.4134/BKMS.b180180
pISSN: 1015-8634 / eISSN: 2234-3016

SEMISYMMETRIC PROPERTIES OF ALMOST COKÄHLER 3-MANIFOLDS

Uday Chand De, Pradip Majhi, and Young Jin Suh

Abstract. In this paper it is proved that on an almost coKähler 3-manifold M, (i) M is h-semisymmetric, (ii) the curvature condition $Q \cdot R = 0$ and (iii) M is coKähler are equivalent.

1. Introduction

In the last several decades, the study of almost contact geometry has been an interesting research field for both of pure mathematical and physical viewpoints. One important class of differentiable manifolds in the framework of almost contact geometry is known as the coKähler manifolds, which were first introduced by Blair [1] and studied by Blair [2], Goldberg and Yanno [6] and Olszak et al. ([4], [8]). The new terminology was recently adopted by many authors mainly due to Li [7], in which the author gave a topological construction of coKähler manifolds via Kähler mapping tori. According to Li's work, we see that the coKähler manifolds are really odd dimensional analogues of Kähler manifolds. We also refer the readers to a recent survey by Cappelletti-Montano et al. [3] and many references therein regarding geometric and topological results on coKähler manifolds.

As a generalization of coKähler manifolds and an analogy of almost Kähler manifolds, the almost coKähler manifolds were widely studied by many authors recently. In particular, D. Perrone [10] obtain a complete classification of homogeneous almost coKähler manifolds of dimension three and also gave a local characterization of such manifolds under a condition of local symmetry. Also, D. Perrone [11] characterized the minimality of the Reeb vector field of three-dimensional almost coKähler manifolds. In addition, a new local classification of three-dimensional almost coKähler manifolds under the condition "$\nabla_\xi h = 2f\phi h$ and $||grad(\lambda)||$ is a non-zero constant, where f is a smooth

Received February 28, 2018; Revised June 5, 2018; Accepted June 21, 2018.
2010 *Mathematics Subject Classification.* 53C15, 53C25.
Key words and phrases. almost coKähler manifold, coKähler manifold, h-semisymmetric manifold, ϕ-symmetric manifold, semisymmetric manifold.
The author Professor Young Jin Suh is supported by Grant Proj. No. NRF-NRF-2018-R1D1A1B-05040381 from National Research Foundation of Korea.

function and λ denotes a positive eigen value function of $h = \frac{1}{2}\pounds_\xi \phi$" was also provided by Erken and Murathan [5].

In a recent paper [16] Wang proves that an almost coKähler 3-manifold is semisymmetric if and only if it is coKähler. Also Wang ([14], [15]) characterizes almost coKähler manifolds satisfying certain curvature conditions. Motivated by the above studies in the present paper we characterize h-semisymmetric almost coKähler 3-manifolds and almost coKähler 3-manifolds satisfying the curvature condition $Q \cdot R = 0$, where R is the Riemannian curvature tensor and Q is the Ricci operator.

A Riemannian or a semi-Riemannian manifold is said to be semisymmetric if

$$R(X, Y) \cdot R = 0$$

holds, where $R(X, Y)$ is the curvature operator. A general study of semisymmetric Riemannian manifolds was made by Szabó [13].

A contact metric manifold is said to be ϕ-semisymmetric if

$$R(X, Y) \cdot \phi = 0$$

holds [17].

An almost coKähler manifold is said to be h-semisymmetric if $R \cdot h = 0$, that is, if

$$(R(X, Y) \cdot h)(Z) = 0$$

for all vector fields X, Y, Z, where $h = \frac{1}{2}\pounds_\xi \phi$.

An example of a curvature condition of semisymmetry type is the following

$$Q \cdot R = 0, \tag{1}$$

where Q is the Ricci operator defined by $S(X, Y) = g(QX, Y)$ and $Q \cdot R$ is defined by

$$(Q \cdot R)(X, Y)Z = Q(R(X, Y)Z) - R(QX, Y)Z - R(X, QY)Z - R(X, Y)QZ$$

for all smooth vector fields X, Y, Z.

A natural extension of such curvature conditions form curvature conditions of pseudosymmetry type. The curvature condition $Q \cdot R = 0$ have been studied by Verheyen et al. in [12].

In the present paper we obtain an equivalent condition under which an almost coKählar manifold to be a co-Kählar manifold. More precisely the following theorem is proved:

Theorem 1.1. *In an almost coKähler 3-manifold M the following conditions are equivalent*:
(i) *M is h-semisymmetric.*
(ii) *M satisfies $Q \cdot R = 0$.*
(iii) *M is coKähler.*

2. Almost coKähler 3-manifolds

By an almost contact metric structure [2] defined on a smooth differentiable manifold M^{2n+1} of dimension $2n+1$ we mean a structure (ϕ, ξ, η, g) satisfying

$$\phi^2 = -id + \eta \otimes \xi, \quad \eta(\xi) = 1, \tag{2}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{3}$$

for all vector field X and Y, where ϕ is a $(1,1)$-type tensor field, ξ is a vector field called the Reeb vector field and η is a 1-form called the almost contact 1-form and g is the Riemannian matric called compatible metric with respect to the almost contact structure.

In the present paper, by an almost coKähler manifold we mean an almost contact metric manifold $(M^{2n+1}, \phi, \xi, \eta, g)$ in which η and Φ are closed, where the fundamental 2-form Φ of the almost contact metric manifold M^{2n+1} is defined by

$$\Phi(X, Y) = g(X, \phi Y)$$

for all vector fields X and Y.

We consider the product $M^{2n+1} \times \mathbb{R}$ of an almost contact metric manifold M^{2n+1} and $\mathbb{R}$ and define on it an almost contact structure J by

$$J(X, f\frac{d}{dt}) = (\phi X - f\xi, \eta(X)\frac{d}{dt}),$$

where X denotes a vector field tangent to M^{2n+1}, t is the coordinate of $\mathbb{R}$ and f is a $\mathcal{C}^{\infty}$-function on $M^{2n+1} \times \mathbb{R}$. We denote by $[\phi, \phi]$ the Nijenhuis tensor of ϕ. If

$$[\phi, \phi] = -2d\eta \otimes \xi$$

holds, or equivalently, J is integrable, then the almost contact metric structure is said to be normal. A normal almost coKähler manifold is called a coKähler manifold.

On an almost coKähler manifold $(M^{2n+1}, \phi, \xi, \eta, g)$ we set $h = \frac{1}{2}\pounds_{\xi}\phi$, where $\pounds$ is the Lie differentiation. We consider the Jacobi operator $l = R(\cdot, \xi)\xi$ generated by ξ and define $h' = h \circ \phi$, where R is the Riemannian curvature tensor of g. We know that ([2], [8], [9]) the three $(1,1)$-type tensor fields l, h' and h are symmetric and satisfy $h\xi = 0$, $l\xi = 0$, $trh = 0$, $tr(h') = 0$ and $h\phi + \phi h = 0$ and

$$\nabla \xi = h'. \tag{4}$$

We denote by $\mathcal{D}$ the distribution $\mathcal{D} = ker\eta$ which is of dimension $2n$. It is easy to check that each integral manifold of $\mathcal{D}$, with the restriction of ϕ, admits an almost Kähler structure. If the associated almost Kähler structure is integrable, or equivalently,

$$(\nabla_X \phi)Y = g(hX, Y)\xi - \eta(Y)hX \tag{5}$$

for any vector fields X, Y, then M^{2n+1} is said to have Kählerian leaves. We see that an almost coKähler manifold is coKähler if and only if [1]

$$\nabla\phi = 0 (\Leftrightarrow \nabla\Phi = 0). \tag{6}$$

Therefore, it follows directly that a 3-dimensional almost coKähler manifold is coKähler if and only if h vanishes ([8]).

3. Some results on almost coKähler 3-manifolds

Let M be an almost coKähler 3-manifolds. Let U_1 be the open subset of M satisfying $h \neq 0$ and U_2 be the open subset of M defined by U_2=$\{p \in M : h = 0$ in a neighborhood of $p\}$. Therefore $U_1 \bigcup U_2$ is an open and dence subset of M and there exists a local orthonormal basis $\{\xi, e, \phi e\}$ of three smooth unit eigen vectors of h for any point $p \in U_1 \bigcup U_2$. On U_1, we set $he = \lambda e$ and hence $h\phi e = -\lambda\phi e$, where λ is a positive eigen value function on U_1. Note that λ is continuous on M and smooth on $U_1 \bigcup U_2$.

Lemma 3.1 ([11])**.** *On U_1, the Levi-Civita connection is given by*

$$\nabla_\xi\xi = 0, \quad \nabla_\xi e = a\phi e, \quad \nabla_\xi\phi e = -ae, \quad \nabla_e\xi = -\lambda\phi e, \quad \nabla_{\phi e}\xi = -\lambda e,$$
$$\nabla_e e = \frac{1}{2\lambda}(\phi e(\lambda) + \sigma(e))\phi e, \quad \nabla_{\phi e}\phi e = \frac{1}{2\lambda}(e(\lambda) + \sigma(\phi e))e,$$
$$\nabla_{\phi e}e = \lambda\xi - \frac{1}{2\lambda}(e(\lambda) + \sigma(\phi e))\phi e, \quad \nabla_e\phi e = \lambda\xi - \frac{1}{2\lambda}(\phi e(\lambda) + \sigma(e))e,$$

where a is a smooth function and σ is the 1-form defined by $\sigma(\cdot) = S(\cdot, \xi)$.

The Ricci operator Q of an almost coKähler 3-manifold is expressed on U_1 by [11]

$$Q\xi = -2\lambda^2\xi + \sigma(e)e + \sigma(\phi e)\phi e, \tag{7}$$

$$Qe = \sigma(e)\xi + \frac{1}{2}(r + 2\lambda^2 - 4\lambda a)e + \xi(\lambda)\phi e, \tag{8}$$

$$Q\phi e = \sigma(\phi e)\xi + \xi(\lambda)e + \frac{1}{2}(r + 2\lambda^2 + 4\lambda a)\phi e, \tag{9}$$

with respect to the local basis $\{\xi, e, \phi e\}$, where r denotes the scalar curvature.

Now, it is well known that the curvature tensor R of any Riemannian 3-manifold is given by

$$R(X,Y)Z = g(Y,Z)QX - g(X,Z)QY + g(QY,Z)X - g(QX,Z)Y$$
$$- \frac{r}{2}[g(Y,Z)X - g(X,Z)Y]$$

for any vector fields X, Y, Z.

Applying the relations (7)-(9), in the above expression we obtain the curvature tensor R of a non-coKähler almost coKähler 3-manifold M as the following:

$$R(e,\xi)\xi = -\lambda(\lambda + 2a)e + \xi(\lambda)\phi e, \tag{10}$$

$$R(\phi e,\xi)\xi=\xi(\lambda)e-\lambda(\lambda-2a)\phi e, \tag{11}$$

$$R(e,\xi)e=\lambda(\lambda+2a)\xi-\sigma(\phi e)\phi e, \tag{12}$$

$$R(e,\xi)\phi e=-\xi(\lambda)\xi+\sigma(\phi e)e, \tag{13}$$

$$R(\phi e,\xi)e=-\xi(\lambda)\xi+\sigma(e)\phi e, \tag{14}$$

$$R(\phi e,\xi)\phi e=\lambda(\lambda-2a)\xi-\sigma(e)e, \tag{15}$$

$$R(e,\phi e)\xi=\sigma(\phi e)e-\sigma(e)\phi e, \tag{16}$$

$$R(e,\phi e)e=-\sigma(\phi e)\xi-(\frac{r}{2}+2\lambda^2)\phi e, \tag{17}$$

$$R(e,\phi e)\phi e=\sigma(e)\xi+(\frac{r}{2}+2\lambda^2)e. \tag{18}$$

At the end of this section we present some results which will be used in the last section:

Theorem 3.2 ([16]). *An almost coKähler 3-manifold is coKähler if and only if it is semisymmetric.*

Theorem 3.3 ([6]). *An almost coKähler manifold is coKähler if and only if $R\cdot\phi=\phi\cdot R$, that is, the manifold is ϕ-semisymmetric.*

4. *h*-semisymmetric almost coKähler 3-manifolds

In this section we prove the following:

Proposition 4.1. *An almost coKähler 3-manifold is h-semisymmetric if and only if it is coKähler.*

To prove the Proposition we first state and prove the following:

Lemma 4.2. *A non-coKähler almost coKähler 3-manifold is h-semisymmetric if and only if the following equations hold*:

$$\lambda\sigma(\phi e)=0, \tag{19}$$

$$\lambda(r+4\lambda^2)=0, \tag{20}$$

$$\lambda\sigma(e)=0, \tag{21}$$

$$\lambda^2(\lambda+2a)=0, \tag{22}$$

$$\lambda\xi(\lambda)=0, \tag{23}$$

$$\lambda^2(\lambda-2a)=0. \tag{24}$$

Proof. Using the equations (10)-(18), after a direct calculation we obtain the following equations:

$$(R(e,\phi e)\cdot h)(e)=\lambda\sigma(\phi e)\xi+\lambda(r+4\lambda^2)\phi e. \tag{25}$$

$$(R(e,\phi e)\cdot h)(\phi e)=2\lambda\sigma(e)\xi+\lambda(r+4\lambda^2)e. \tag{26}$$

$$(R(e,\phi e)\cdot h)(\xi)=\lambda\sigma(\phi e)e+\lambda\sigma(e)\phi e. \tag{27}$$

$$(R(e,\xi)\cdot h)(e)=\lambda^2(\lambda+2a)\xi-2\lambda\sigma(\phi e)\phi e. \tag{28}$$

$$(R(e,\xi)\cdot h)(\phi e)=\lambda\xi(\lambda)\xi-2\lambda\sigma(\phi e)e. \tag{29}$$

$$(R(e,\xi)\cdot h)(\xi)=\lambda^2(\lambda+2a)e+\lambda\xi(\lambda)\phi e. \tag{30}$$

$$(R(\phi e,\xi)\cdot h)(e)=\lambda\xi(\lambda)\xi-2\lambda\sigma(e)\phi e. \tag{31}$$

$$(R(\phi e,\xi)\cdot h)(\phi e)=\lambda^2(\lambda-2a)\xi+2\lambda\sigma(e)e. \tag{32}$$

$$(R(\phi e,\xi)\cdot h)(\xi)=\lambda^2(\lambda-2a)\phi e+\lambda\xi(\lambda)e. \tag{33}$$

Now if the manifold is h-semisymmetic, then the l.h.s of the equations (25)-(33) vanish. Finally, the equations (19)-(24) follow directly from the equations (25)-(33), on the argument that $\{e,\phi e,\xi\}$ is a basis. Conversely, if the conditions (19)-(24) hold, then it is straight forward that the manifold is h-semisymmetic. □

Proof of Proposition 4.1. For a coKähler 3-manifold $h=0$ implies $R(X,Y)\cdot h=0$, that is, the coKähler 3-manifold is h-semisymmetric. Then to complete the proof, we are remains to prove that a non-coKähler almost coKähler 3-manifold can not be h-semisymmetric. For this, let us assume that a non-coKähler almost coKähler 3-manifold M is h-semisymmetric. From (22) and (24) we get $a=0$. Then from (22) we have $\lambda=0$, which contradicts our assumption, that is, M is coKähler. □

5. Almost coKähler 3-manifolds satisfying $Q\cdot R=0$

In the present section, we prove the following:

Proposition 5.1. *An almost coKähler 3-manifold satisfies the curvature condition $Q\cdot R=0$ if and only if it is coKähler.*

To prove the Proposition we first state and prove the following:

Lemma 5.2. *A non-coKähler almost coKähler 3-manifold satisfies the curvature condition $Q \cdot R = 0$ if and only if the following equations hold:*

$$(3r + 8\lambda^2 - 8\lambda a)\sigma(\phi e) = 0, \tag{34}$$

$$\sigma(e)\sigma(\phi e) = 0, \tag{35}$$

$$\sigma(e)\xi(\lambda), \tag{36}$$

$$\sigma(e) = 0, \tag{37}$$

$$(3r + 6\lambda^2 - 4\lambda a)(\frac{r}{2} + 2\lambda^2) = 0, \tag{38}$$

$$(r + 4\lambda^2)\xi(\lambda) = 0, \tag{39}$$

$$(r + 2\lambda^2 + 4\lambda a)\sigma(e) = 0, \tag{40}$$

$$r + 2\lambda^2 + 4\lambda a = 0, \tag{41}$$

$$\xi(\lambda)\sigma(\phi e) = 0, \tag{42}$$

$$\sigma(\phi e) = 0, \tag{43}$$

$$r\sigma(\phi e) = 0, \tag{44}$$

$$\lambda(r + 2\lambda^2 - 4\lambda a)(\lambda + 2a) = 0, \tag{45}$$

$$(r + 2\lambda^2 - 4\lambda a)\sigma(\phi e) = 0, \tag{46}$$

$$\lambda(\lambda + 2a)\sigma(e) = 0, \tag{47}$$

$$\xi(\lambda) = 0, \tag{48}$$

$$(r - 2\lambda^2 - 4\lambda a)\xi(\lambda) = 0, \tag{49}$$

$$(r - 2\lambda^2 - 4\lambda a)\sigma(\phi e) = 0, \tag{50}$$

$$\lambda(r + 2\lambda^2 + 4\lambda a)\sigma(\lambda - 2a) = 0, \tag{51}$$

$$\lambda(\lambda + 2a)\sigma(e) = 0, \tag{52}$$

$$\xi(\lambda)\sigma(\phi e) = 0, \tag{53}$$

$$\lambda^2\xi(\lambda) = 0, \tag{54}$$

$$\lambda^3(\lambda + 2a) = 0, \tag{55}$$

$$\lambda(\lambda - 2a) = 0, \tag{56}$$

$$\lambda(\lambda - 2a)\sigma(\phi e) = 0. \tag{57}$$

Proof. From (10)-(18), after simplification we obtain the following equations:

$$(Q\cdot R)(e,\phi e)e = \frac{1}{2}(3r+8\lambda^2-8\lambda a)\sigma(\phi e)\xi - 2\sigma(e)\sigma(\phi e)e - 2\sigma(e)\xi(\lambda)\xi + 2\sigma(e)^2\phi e + (\frac{3r}{2}+3\lambda^2 - 2\lambda a)(\frac{r}{2}+2\lambda^2)\phi e - \frac{1}{2}(r+4\lambda^2)\xi(\lambda)e. \tag{58}$$

$$(Q\cdot R)(e,\phi e)\phi e = -(r+2\lambda^2+4\lambda a)\sigma(e)\xi + 2\sigma(e)\sigma(\phi e)\phi e - (r+2\lambda^2+4\lambda a)e + (r+4\lambda^2)\xi(\lambda)\phi e + 2\xi(\lambda)\sigma(\phi e)\xi - \sigma(\phi e)^2 e. \tag{59}$$

$$(Q\cdot R)(e,\phi e)\xi = -r\sigma(\phi e)e. \tag{60}$$

$$(Q\cdot R)(e,\xi)e = \lambda(-r-2\lambda^2+4\lambda a)(\lambda+2a)\xi + (r+2\lambda^2 - 4\lambda a)\sigma(\phi e)\phi e + 2\lambda(\lambda+2a)\sigma(e)e - 2\sigma(\phi e)\xi(\lambda)e + 2\xi(\lambda)^2\xi - 2\xi(\lambda)\sigma(e)\phi e. \tag{61}$$

$$(Q\cdot R)(e,\xi)\phi e = (\frac{r}{2}-2\lambda a-\lambda^2)\xi(\lambda)\xi + (\lambda^2+2\lambda a - \frac{r}{2})\sigma(\phi e)e - \frac{1}{2}(r+2\lambda^2+4\lambda a)\lambda(\lambda-2a)\xi + \frac{1}{2}(r+2\lambda^2+4\lambda a)\sigma(e)e. \tag{62}$$

$$(Q\cdot R)(e,\xi)\xi = -2\lambda(\lambda+2a)\sigma(e)\xi + 2\xi(\lambda)\sigma(\phi e)\xi + 4\lambda^2\xi(\lambda)\phi e - 4\lambda^3(\lambda+2a)e - 2\sigma(\phi e)^2 e + 2\sigma(e)\sigma(\phi e)e. \tag{63}$$

$$(Q\cdot R)(\phi e,\xi)e = r[\xi(\lambda)\xi - \sigma(e)\phi e]. \tag{64}$$

$$(Q\cdot R)(\phi e,\xi)\phi e = (r+2\lambda^2+4\lambda a)\sigma(e)e - \lambda(r+2\lambda^2+4\lambda a)(\lambda - 2a)\xi + 2\lambda(\lambda-2a)\sigma(\phi e)\phi e - 2\xi(\lambda)\sigma(e)\phi e + 2\xi(\lambda)^2\xi - 2\xi(\lambda)\sigma(\phi e)e. \tag{65}$$

$$(Q\cdot R)(\phi e,\xi)\xi = 2\xi(\lambda)\sigma(e)\xi - 2\lambda(\lambda-2a)\sigma(\phi e)\xi + 4\lambda^2\xi(\lambda)e - 4\lambda^3(\lambda-2a)\phi e - 2\sigma(e)^2\phi e + 2\sigma(e)\sigma(\phi e)e. \tag{66}$$

□

Now if the manifold satisfies the curvature property $Q\cdot R = 0$, then the l.h.s of the equations (58)-(66) vanish. Then the equations of Lemma 5.1 follows directly from the equations (58)-(66), by the hypothesis that $\{e, \phi e, \xi\}$ is a basis. Conversely, if the equations (34)-(57) hold, then it is clear to state that the manifold satisfies the curvature property $Q\cdot R = 0$.

Proof of Proposition 5.1. For a coKähler 3-manifold, it is well known that the Ricci operator is given by

$$Q = \frac{r}{2}(id - \eta \otimes \xi).$$

This implies directly that a coKähler 3-manifold satisfies $Q \cdot R = 0$. Then to complete the proof, we are remains to prove that an almost coKähler 3-manifold satisfying $Q \cdot R = 0$ is coKähler. For this, let us assume that a non-coKähler almost coKähler 3-manifold M satisfies $Q \cdot R = 0$. In this regard from (55) and (56) we get $a = 0$. Then from (56) we have $\lambda = 0$, which contradicts our assumption, that is, M is cokähler. □

6. Proof of the main Theorem

In Proposition 4.1 we prove that an almost coKähler 3-manifold is coKähler if and only if $R \cdot h = 0$ and in Proposition 5.1 we prove that an almost coKähler 3-manifold is coKähler if and only if $Q \cdot R = 0$. In view of Proposition 4.1 and Proposition 5.1 we have our main Theorem 1.1.

Remark 6.1. Observe that the conditions of Theorem 1.1 are equivalent to "M is semisymmetric" (Wang [16]) and to "M is ϕ-semisymmetric" (Goldberg and Yano [6]).

Acknowledgement. The authors are thankful to the Referee for his/her valuable suggestions and comments towards the improvement of the paper.

References

[1] D. E. Blair, *The theory of quasi-Sasakian structures*, J. Differential Geometry **1** (1967), 331–345.
[2] ———, *Riemannian Geometry of Contact and Symplectic Manifolds*, second edition, Progress in Mathematics, **203**, Birkhäuser Boston, Inc., Boston, MA, 2010.
[3] B. Cappelletti-Montano, A. De Nicola, and I. Yudin, *A survey on cosymplectic geometry*, Rev. Math. Phys. **25** (2013), no. 10, 1343002, 55 pp.
[4] P. Dacko and Z. Olszak, *On almost cosymplectic (κ, μ, ν)-spaces*, in PDEs, submanifolds and affine differential geometry, 211–220, Banach Center Publ., **69**, Polish Acad. Sci. Inst. Math., Warsaw, 2005.
[5] K. Erken and C. Murathan, *A class of 3-dimensional almost cosymplectic manifolds*, Turkish J. Math. **37** (2013), no. 5, 884–894.
[6] S. I. Goldberg and K. Yano, *Integrability of almost cosymplectic structures*, Pacific J. Math. **31** (1969), 373–382.
[7] H. Li, *Topology of co-symplectic/co-Kähler manifolds*, Asian J. Math. **12** (2008), no. 4, 527–543.
[8] Z. Olszak, *On almost cosymplectic manifolds*, Kodai Math. J. **4** (1981), no. 2, 239–250.
[9] ———, *Locally conformal almost cosymplectic manifolds*, Colloq. Math. **57** (1989), no. 1, 73–87.
[10] D. Perrone, *Classification of homogeneous almost cosymplectic three-manifolds*, Differential Geom. Appl. **30** (2012), no. 1, 49–58.
[11] ———, *Minimal Reeb vector fields on almost cosymplectic manifolds*, Kodai Math. J. **36** (2013), no. 2, 258–274.

[12] P. Verheyen and L. Verstraelen, *A new intrinsic characterization of hypercylinders in Euclidean spaces*, Kyungpook Math. J. **25** (1985), no. 1, 1–4.
[13] Z. Szabó, *Structure theorems on Riemannian spaces satisfying* $R(X, Y) \cdot R = 0$. *I*, The local version, J. Differential Geom. **17** (1982), no. 4, 531–582 (1983).
[14] Y. Wang, *Almost co-Kähler manifolds satisfying some symmetry conditions*, Turkish J. Math. **40** (2016), no. 4, 740–752.
[15] ______, *Ricci tensors on three-dimensional almost coKähler manifolds*, Kodai Math. J. **39** (2016), no. 3, 469–483.
[16] ______, *Semi-symmetric almost coKähler* 3*-manifolds*, Internat. J. Geom. Methods in Modern Phys. **15** (2018), 1850031, 12pp.
[17] A. Yildiz and U. C. De, *A classification of* (k, μ)*-contact metric manifolds*, Commun. Korean Math. Soc. **27** (2012), no. 2, 327–339.

Uday Chand De
Department of Pure Mathematics
University of Calcutta
35, Ballygaunge Circular Road
Kolkata-700019, West Bengal, India
Email address: uc_de@yahoo.com

Pradip Majhi
Department of Pure Mathematics
University of Calcutta
35, Ballygaunge Circular Road
Kolkata-700019, West Bengal, India
Email address: mpradipmajhi@gmail.com

Young Jin Suh
Department of Mathematics
Kyungpook National University
Taegu 702-701, Korea
Email address: yjsuh@knu.ac.kr